

Minot State University

Math Track Meet

Grades 7-8

April 22, 2009

Event I:

Number Theory & Operations (No Calculator)

Mark the appropriate answer on the scantron card.

- To number the pages of a book from 1 to 10, one must write 11 digits. How many digits must one write to number pages of a book from 1 to 456?
A) 1167 B) 1257 C) 1260 D) 1368 E) none of the previous
- What is the 147th odd natural number?
A) 293 B) 295 C) 297 D) 299 E) none of the previous
- What is the arithmetic mean of the first fifteen prime numbers?
A) $19.\bar{3}$ B) $20.5\bar{3}$ C) $21.\bar{3}$ D) $21.8\bar{6}$ E) none of the previous
- If $a \blacklozenge b = a + b - ab$, for what integer(s) x is $x \blacklozenge x = 0$?
A) 0 and 1 B) -2 and 2 C) 0 and 2 D) -1 and 1 E) none of the previous
- Which of the following is equal to 59?
A) 63_{Eight} B) 110101_{Two} C) 213_{Five} D) 49_{Twelve} E) none of the previous
- If the time is 10:05, what time will it be in 1100 minutes?
A) 9:05 B) 4:20 C) 9:15 D) 4:25 E) none of the previous

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Event II:

Fractions, Decimals, Percents (No Calculator)

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7. If $x = \frac{1}{2}$ and $y = \frac{2}{3}$, find the value of $\left(\frac{1}{x} + \frac{1}{y}\right) + \frac{x}{y}$ as a mixed number in simplest form.
- A) $1\frac{1}{2}$ B) $4\frac{1}{4}$ C) $\frac{2}{3}$ D) $3\frac{5}{6}$ E) none of the previous
8. Which of the following fractions, when written as a decimal, will not terminate: $\frac{3}{150}$, $\frac{7}{75}$, $\frac{11}{250}$, $\frac{15}{48}$?
- A) $\frac{3}{150}$ B) $\frac{7}{75}$ C) $\frac{11}{250}$ D) $\frac{15}{48}$ E) none of the previous
9. There are 36 students in a pre-algebra class. Two-thirds of the students are boys, and one-fourth of the boys are five feet six inches tall or taller. If all the girls in the class are less than five feet six inches tall, what fraction of the whole class is less than five feet six inches tall?
- A) $\frac{2}{3}$ B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) $\frac{5}{6}$ E) none of the previous
10. A printer can be set to print 12 characters in 1 inch or 10 characters in 1 inch. How many more characters can be put on a $7\frac{1}{2}$ inch line if the more compact of the two settings is used instead of the other setting?
- A) 10 B) 12 C) 15 D) 18 E) none of the previous
11. A theorem in Number Theory states that the Nth prime number is approximately $N\left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{N}\right)$. Determine the decimal form of the absolute value of the difference between the 6th prime and this approximation of the 6th prime.
- A) 1.6 B) 1.7 C) 1.8 D) 1.9 E) none of the previous
12. On a number line the coordinate of Point A is $-8\frac{1}{2}$ and the coordinate of Point B is $11\frac{1}{2}$. What is the coordinate of Point C if the distance between A and C is one-third of the distance between B and C?
- A) $-18\frac{1}{2}, -3\frac{1}{2}$ B) $6\frac{1}{2}, 16\frac{1}{2}$ C) $-14\frac{5}{6}, -2\frac{1}{6}$ D) $5\frac{1}{6}, 17\frac{5}{6}$ E) none of the previous

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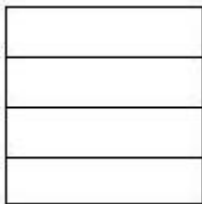
Grades 7-8

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Event III:

Informal Geometry

Mark the appropriate answer on the scantron card.

1. A right triangle has a side length of 21 inches and a hypotenuse of 29 inches. A second triangle is similar to the first and has a hypotenuse of 87 inches. What is the length of the shortest side of the second triangle?
- A) 20 in. B) 60 in. C) 62 in. D) 63 in. E) none of the previous
2. Point B has the ordered pair $(2, -4)$. Point B was reflected through the y -axis to obtain the image B' . B' was translated by moving eight units up and six units right to obtain the point B'' . What are the coordinates of point B'' ?
- A) $(-8, 4)$ B) $(8, 12)$ C) $(4, 4)$ D) $(2, 4)$ E) none of the previous
3. The square shown to the right is divided into 4 congruent (identical) rectangles. If the perimeter of the square is 96 units, what is the perimeter of one of the four congruent rectangles?
- A) 36 units B) 48 units C) 60 units D) 72 units
- E) none of the previous
- 
4. A sheet of paper measures 8.5 inches by 11 inches. A one-inch border of paper is cut off from each of the four sides. How many square inches have been cut off? Give your answer to the nearest tenth.
- A) 26 in^2 B) 26.5 in^2 C) 35 in^2 D) 58.5 in^2 E) none of the previous
5. A polygon is said to be regular if it is equilateral and equiangular. What is the degree measure of an exterior angle of a regular pentagon?
- A) 36° B) 54° C) 72° D) 108° E) none of the previous
6. Twenty-one units is the shortest leg of a right triangle whose other leg and hypotenuse are consecutive whole numbers. What is the sum of all three sides of the right triangle?
- A) 220 units B) 221 units C) 441 units D) 462 units E) none of the previous

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Event IV:

Problem Solving, Probability, and Statistics

Mark the appropriate answer on the scantron card.

1. A standard six-sided die was rolled 50 times, and the outcomes are in the table shown at the right. What percent of the rolls resulted in a prime number?

Outcome	# of Occurrences
1	12
2	9
3	8
4	10
5	5
6	6

- A) 22% B) 34% C) 44% D) 68%
E) none of the previous

2. At the Alphabet Store, each vowel (a, e, i, o, u) sells for a different price, but all consonants are free. The word “vertex” sells for \$4, “rhombi” sells for \$13, “geometry” sells for \$16. How much would the word “trigonometry” cost?

- A) \$17 B) \$23 C) \$25 D) \$27 E) none of the previous

3. The median of a set of consecutive even integers is 133. If the smallest integer in the set is 126, what is the greatest integer in the set?

- A) 138 B) 140 C) 142 D) 150 E) none of the previous

4. A local ice cream shop carries vanilla ice cream, chocolate ice cream, and cookies-n-cream ice cream. It also carries sugar, wafer, and waffle cones. Customers can order a single or double scoop of their favorite flavor, or they can choose one scoop of one flavor topped with one scoop of another flavor. If vanilla topped with chocolate is the same as chocolate topped with vanilla, how many different ice cream cone orders are possible?

- A) 18 orders B) 21 orders C) 24 orders D) 27 orders E) none of the previous

5. A car is scheduled to make a 728-mile trip in 10 hours. The car averages 70 miles per hour (mph) during the first 350 miles and 75 mph during the next 150 miles. What must the average speed of the car be for the remainder of the trip in order for the car to arrive on schedule?

- A) 57 mph B) 65 mph C) 74 mph D) 76 mph E) none of the previous

6. The stem-and-leaf plot below shows the scores of 25 students. What percent of the students scored less than 85%? (The value 4 | 8 represents 48%.)

4		8 9
5		1 4 7
6		0 5 6 6 7 8
7		2 3 5 5 8 9
8		4 5 6 7 9
9		2 2 5

- A) 66% B) 72% C) 76% D) 82% E) none of previous

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Event I: Number Theory and Operations (No Calculators)

1. C The first nine pages each have one-digit page numbers. Of the first 99 pages, the 90 pages after the first 9 each require two digits. After the first 99 pages, the remaining pages, $456 - 99$ or 357 pages, each require 3 digits. The total number of digits required to write all 456 page numbers will then be $1 \cdot 9 + 2 \cdot 90 + 3 \cdot 357 = 9 + 180 + 1071 = 1260$, which is answer C.
2. A If we let k be a natural number, the odd natural numbers can be written in the form $2k - 1$. Using this formula, when $k = 1, 2,$ and 3 respectively, $2k - 1 = 1, 3,$ and 5 . The value k then provides a count of the odd natural numbers. Thus when $k = 147$, we find that the 147th odd natural number is $2 \cdot 147 - 1 = 294 - 1 = 293$, which is answer A.
3. D The arithmetic mean of the first 15 prime numbers is found by dividing the sum of the numbers by 15. This would yield $(2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 + 23 + 29 + 31 + 37 + 41 + 43 + 47) / 15 = 328 / 15 = 21.\overline{86}$, which is answer D.
4. C To find the values that make $x \blacklozenge x = 0$, we will replace both a and b with x in the operation rule $a \blacklozenge b = a + b - ab$. This gives $x \blacklozenge x = x + x - x^2$. Then $0 = x + x - x^2 = 2x - x^2 = x(2 - x)$. Using the zero product property, either $x = 0$ or $2 - x = 0$. This gives 2 solutions: 0 and 2, which are given as answer C.
5. E $63_{\text{Eight}} = 6 \cdot 8^1 + 3 \cdot 1 = 48 + 3 = 51$. Also, $110101_{\text{Two}} = 1 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 1 = 53$. Next $213_{\text{Five}} = 2 \cdot 5^2 + 1 \cdot 5^1 + 3 \cdot 1 = 58$, and $49_{\text{Twelve}} = 4 \cdot 12^1 + 9 \cdot 1 = 57$. None of these is equal to 59. Therefore the answer is E, none of the previous choices.
6. D The number of hours equivalent to 1100 minutes is $1100 / 60 = 18\frac{1}{3}$ hours. In 12 hours, the time will again be 10:05, and there will be $6\frac{1}{3}$ hours remaining to pass. In two hours, the time will be 12:05, with $4\frac{1}{3}$ hours remaining. When the additional $4\frac{1}{3}$ hours passes, it will be 4:25, since $1/3$ of an hour is 20 minutes. Therefore the time will be 4:25, which is answer D.

Grades 7 & 8
Math Track Meet Solutions
April 22, 2009

Event II: Fractions, Decimals, Percents (No Calculators)

1. B Substituting $x = \frac{1}{2}$ and $y = \frac{2}{3}$ into $\left(\frac{1}{x} + \frac{1}{y}\right) + \frac{x}{y}$, we get

$$\left(\frac{1}{\frac{1}{2}} + \frac{1}{\frac{2}{3}}\right) + \frac{\frac{1}{2}}{\frac{2}{3}} = (1 \div \frac{1}{2} + 1 \div \frac{2}{3}) + \frac{1}{2} \div \frac{2}{3} = (1 \cdot \frac{2}{1} + 1 \cdot \frac{3}{2}) + \frac{1}{2} \cdot \frac{3}{2} = 2 + 1\frac{1}{2} + \frac{3}{4} = 3\frac{5}{4} = 4\frac{1}{4} \text{ which is answer B.}$$

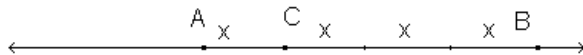
2. B A decimal will terminate if its reduced form has a denominator that has a prime factorization composed only of 2's and/or 5's. Reducing $\frac{3}{150}$, $\frac{7}{75}$, $\frac{11}{250}$, $\frac{15}{48}$, we get $\frac{1}{50} = \frac{1}{2 \cdot 5^2}$, $\frac{7}{75} = \frac{7}{3 \cdot 5^2}$, $\frac{11}{250} = \frac{11}{2 \cdot 5^3}$, and $\frac{5}{16}$ respectively. Thus only $\frac{7}{75}$ has a denominator composed of factors other than 2 and 5. Therefore, B is the only value that will not terminate as a decimal value.

3. D If $\frac{2}{3}$ of the 36 students are boys, then the number of boys is 24. Additionally $\frac{1}{4}$ of the boys (6 boys) are five feet six inches tall or taller and the rest (18 boys) are shorter than five feet six inches in height. $\frac{1}{3}$ of the class or 12 students are girls, all of whom are under five feet six inches in height. Thus the number of students under 5 feet 6 inches is 18 boys plus 12 girls or 30 students. This means that $\frac{30}{36}$ or $\frac{5}{6}$ of the students are less than five feet six inches tall, which is answer D.

4. C There are 2 additional characters printed per inch when using the more compact font. In $7\frac{1}{2}$ inches, there would be $7\frac{1}{2} \cdot 2$ or 15 additional characters can be put on a $7\frac{1}{2}$ inch line, which is answer C.

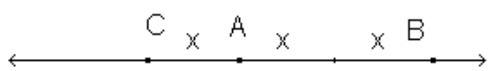
5. B Using the formula $N\left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{N}\right)$ to approximate the 6th prime number, we let $N = 6$. Then $6\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}\right) = 6\left(\frac{60}{60} + \frac{30}{60} + \frac{20}{60} + \frac{15}{60} + \frac{12}{60} + \frac{10}{60}\right) = 6\left(\frac{147}{60}\right) = \frac{147}{10} = 14.7$. The 6th prime is 13, so the absolute difference between 14.7 and 13 is 1.7, which is shown as answer B. Thus B is the correct answer.

6. A Point C may lie either between points A and B or on the side of point A opposite from point B. If point C lies between points A and B, and the distance between A and C is one-third of the distance between B and C, the segment with ends at A and B can be divided into four equal lengths as shown.



Points A and B are 20 units apart, with the coordinate of point A being $-8\frac{1}{2}$ and the coordinate of B being $11\frac{1}{2}$. Then $4x = 20$ and $x = 5$, so the coordinate of point C would be $-3\frac{1}{2}$.

If C lies on the side of point A opposite from point B, three equal lengths will be formed as shown below:



. Then $2x = 20$ and $x = 10$, so the coordinate of point C would be $-18\frac{1}{2}$.

Those values are given as answer A.

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Event III: Informal Geometry

1. B Using proportions and the fact that the triangles are similar, the following proportion can be used to solve for one of the unknown sides, b , of the second triangle.

$$\frac{21}{29} = \frac{b}{87}$$
$$29b = 1827$$
$$b = 63$$

Now it is not known if $b = 63$ is the shortest side of the second right triangle. Using the Pythagorean Theorem, we get:

$$a^2 + 63^2 = 87^2$$

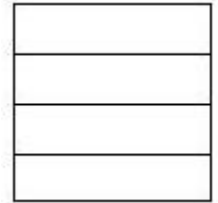
$$a^2 = 3600$$

Thus, 60 in. is the shortest side of the second triangle.

$$a = 60$$

2. C Starting with point $B(2, -4)$, its reflection through the y -axis maintained the same height (y -coordinate) but changed the sign on the x -coordinate, creating the image $B'(-2, -4)$. When B' was translated by moving eight units up and six units right, $(-2 + 6, -4 + 8)$, it created the point $B''(4, 4)$.

3. C The perimeter of the square is 96 units and thus, each side of the square is $96/4 = 24$ units. Dividing a side by four to get the short side of each rectangle, we get $24/4 = 6$ units. Each rectangle is 24 units \times 6 units and therefore has a perimeter of $24 + 24 + 6 + 6 = 60$ units.



4. C Starting with an 8.5" \times 11" sheet of paper and cutting off a one-inch border from each side leaves a rectangle that is 6.5" \times 9". Taking the difference of the area of the original sheet of paper and remaining rectangle after the one-inch borders have been removed will yield the area that was cut off.

$$(8.5" \times 11") - (6.5" \times 9") = 93.5in^2 - 58.5in^2 = 35in^2.$$

5. C Starting with a regular polygon and drawing a line from the center to each vertex creates 5 congruent isosceles triangles inside the pentagon, each with central angle measure of $360^\circ/5 = 72^\circ$. Thus, the measure of the other two angles in each triangle totals $(180^\circ - 72^\circ) = 108^\circ$, meaning each of these angles is 54° . Since two of these angles (108°) make up an interior angle of the pentagon, the exterior angle is $180^\circ - 108^\circ = 72^\circ$.

6. D One leg of the triangle is 21 units and the other leg and hypotenuse are consecutive whole numbers, say x and $(x + 1)$, respectively. Since the triangle is right, we can use the Pythagorean Theorem to solve for the other two sides:

$$21^2 + x^2 = (x+1)^2$$

$$21^2 + x^2 = x^2 + 2x + 1$$

$$440 = 2x$$

Thus, the sum of the sides is $21 + 220 + 221 = 462$ units.

$$x = 220$$

$$x + 1 = 221$$

Grades 7 & 8
Math Track Meet Solutions
April 22, 2009

Event IV: Problem Solving, Probability, and Statistics

1. C The numbers 2, 3, and 5 are prime numbers, which were $9 + 8 + 5 = 22$ of the rolls. Thus, $22/50 = 44/100$ or 44% of the rolls of the die.
2. D Since all consonants are free and the word "vertex" sells for \$4, each "e" is worth \$2. The word "geometry" has one "o" and two "e"s and sells for \$16 which means the "o" is worth $\$16 - \$2 - \$2 = \12 . The word "rhombi" has an "o" and an "i", which means the "i" is worth $\$13 - \$12 = \$1$. Hence, the word "trigonometry" costs $\$1 + \$12 + \$12 + \$2 = \$27$.

Alternate Solution:

vertex: $e + e = \$4$

geometry: $e + e + o = \$16$

rhombi: $o + i = \$13$

Thus, $e = \$2$, $o = \$12$, and $i = \$1$. The word "trigonometry" would cost

$$i + o + o + e = \$1 + \$12 + \$12 + \$2 = \$27.$$

$$2i + a + e = \$6 + \$4 + \$6 = \$16.$$

3. B The smallest member of the set is 126 and the median is 133. Since the set only contains consecutive even numbers and the median need not be a member of the set, the members of the set less than 133 are 126, 128, 130, and 132, and thus, the even integers greater than 133 are 134, 135, 138, and 140. Hence, the greatest integer in the set is 140.
4. D Since the shop sells three flavors of ice cream and three types of cones, there are $3 \times 3 = 9$ different cones with a single scoop of ice cream (vanilla/sugar, vanilla/wafer, vanilla/waffle, ..., cookies-n-cream/waffle). Similarly, there are 9 different double scoop cones in which both scoops are the same flavor. Since vanilla topped with chocolate is the same as chocolate topped with vanilla, there are 3 combinations of different flavored double scoops – vanilla/chocolate, vanilla/cookies-n-cream, and chocolate/cookies-n-cream. Each of these combinations has 3 cone options, for a total of 9 double scoop cones in which each scoop is a different flavor. Thus, there are a total of $9 + 9 + 9 = 27$ different ice cream cone orders possible.
5. D A car is scheduled to make a 728-mile trip in 10 hours. Since the car averaged 70 miles per hour (mph) during the first 350 miles, it traveled for $350 \text{ mi}/70 \text{ mph} = 5$ hours. During the next 150 miles it averaged 75 mph, which means it traveled for 2 hours. Thus, there are 3 hours left to travel the remaining 228 miles. Therefore, the average speed of the car must be $228 \text{ mi}/3 \text{ hours} = 76 \text{ mph}$ for the remainder of the trip in order for the car to arrive on schedule.

6. B Stem-and-leaf plot:

4 | 8 9

5 | 1 4 7

6 | 0 5 6 6 7 8

7 | 2 3 5 5 8 9

8 | 4 5 6 7 9

9 | 2 2 5

Since there are 25 scores and each number on the left represents one student's score, 18 of the 25 scores are less than 85% (48%, 49%, 51%, 54%, ..., 78%, 79%, 84%). Thus, the percent of students who scored less than 85% is $18/25 = 72/100$ or 72%.

Grades 7 & 8
Math Track Meet Solutions
April 22, 2009

Team Event

1. Answer: -6

We are given the four points $A(-6,0)$, $B(0,-6)$, $X(0,10)$, and $Y(16,k)$ on the Cartesian plane. We know segment AB is parallel to segment XY . Thus, the goal is to determine the slope of segment AB and determine k such that segment XY has the same slope (i.e., is parallel).

The slope of segment AB is $m_{AB} = \frac{-6-0}{0-(-6)} = \frac{-6}{6} = -1$. Using this slope we can find k as follows:

$$m_{XY} = \frac{k-10}{16-0} = -1$$

$$k-10 = -16$$

$$k = -6$$

2. Answer: 252 walleye

Let x = the estimate of the number of walleye in the lake. We expect the ratio of total tagged walleye to total walleye in the lake to be in the same proportion as tagged walleye and total walleye in a day's catch. Thus, the following proportions can be used to estimate the number of walleye in the lake.

$$\frac{132}{x} = \frac{22}{42}$$

$$22x = 5544$$

$$x = 252 \text{ walleye}$$

3. Answer: 100 miles

Cyle Cyclist rode his bike a total of 450 miles in 5 days. Each day he rode 5 more miles than he had ridden on the previous day. Let x = the number of miles Cyle rode on day 1. Thus, $(x+5)$, $(x+10)$, $(x+15)$, and $(x+20)$ represent the number of miles Cyle rode on days 2, 3, 4, and 5, respectively. Setting up an algebraic equation in which the sum of the mileage for days 1-5 totals 450 miles, we get:

$$x + (x+5) + (x+10) + (x+15) + (x+20) = 450$$

$$5x + 50 = 450$$

$$5x = 400$$

$$x = 80$$

Thus, Cyle rode $80 + 20 = 100$ miles on day 5.

4. Answer: 936 in^2 (or 936)

The volume of a rectangular prism is $V = lwh$. Since the rectangular base is 6 inches by 18 inches, the height of the prism is

$$h = \frac{1620 \text{ in}^3}{6 \text{ in} \cdot 18 \text{ in}} = 15 \text{ in}. \text{ The total surface area is the sum of the areas of the six faces:}$$

$$SA = 2(6 \text{ in} \cdot 18 \text{ in}) + 2(6 \text{ in} \cdot 15 \text{ in}) + 2(18 \text{ in} \cdot 15 \text{ in}) = 936 \text{ in}^2$$

5. Answer: 18,750

Since 5000 gold pieces were unburied and kept by the pirate, they represent $\frac{2}{3}$ of the coins that were not buried. Then 2500 represents the $\frac{1}{3}$ of the unburied gold pieces that were given to the crew. Therefore, 7500 gold pieces were not buried. They represent the $\frac{2}{5}$ of the treasure that was not buried. This means that 3750 gold coins is $\frac{1}{5}$ of the treasure, and the entire treasure is 5 times 3750 gold coins, or a total of 18,750 gold coins.

6. Answer: 20

With the two requirements that the two whole numbers sum to 99 and be formed with only 2 different digits, pairs of numbers must contain only digits that have a sum of 9. These numbers can be paired to make two 2-digit numbers that include 00 and 99, 11 and 88, 22 and 77, 33 and 66, 44 and 55, 09 and 90, 18 and 81, 27 and 72, 26 and 63, and 45 and 54. This makes 10 ordered pairs that meet the two conditions, where the first number is x and the second is y . 10 more ordered pairs can be made by switching the values so the one listed second in each pair above becomes a different ordered pair. As a result, there will be 20 ordered pairs that satisfy the two conditions.